PHY489 Week 12:
Weak Interactions (Charged, Neutral)

(GWS)

CHARGED WEAK INTERACTIONS

- Mediated by $W^{\pm}$ boson
- Vertex couplings are different

(QED) $\quad i g_e \gamma^{\mu}$ $\quad$ Vector

Parity is maximally violated!

Charged Weak $\quad -i \frac{g_w}{2\sqrt{2}} \gamma^{\mu}(1-\gamma^5)$ $\quad$ Vector − axial Vector "V-A coupling"

Neutral Weak $\quad -i \frac{g_z}{2} \gamma^{\mu}(c_V^f - c_A^f \gamma^5)$ $\quad$ mix of Vector, axial vector

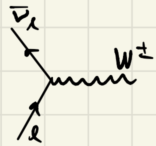
- $W^{\pm}$, Z^0 propagator (vector)

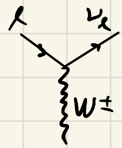
$$-\frac{i}{q^2 - M_{W/Z}^2 c^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_{W/Z}^2 c^2}\right)$$

$$\underset{q^2 \ll M_{W/Z}^2 c^2}{\simeq} \frac{i g_{\mu\nu}}{M_{W/Z}^2 c^2}$$

(Reduced propagator)

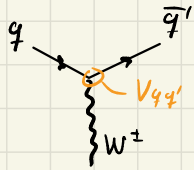
- For leptonic interaction vertices, ie.





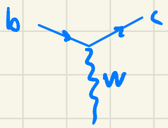
we can neglect neutrino oscillations (for now)

- For quark interaction vertices, ie.



we must include a CKM matrix element vertex factor.

ex. for

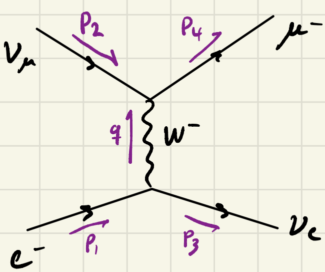


$-\frac{i g_w}{2\sqrt{2}} V_{cb} \gamma^\mu (1-\gamma^5)$ is vertex factor

Inverse Muon Decay

"Inverse muon decay"

- Assume $q^2 \ll M_w^2 c^2$; consider $e^- \nu_\mu \to \mu^- \nu_e$ process



Amplitude is (use reduced propagator)

$$\mathcal{M}_{fi} = \frac{g_w^2}{8(M_w c)^2} \left[\bar{u}(3)\gamma^\mu(1-\gamma^5)u(1)\right]\left[\bar{u}(4)\gamma_\mu(1-\gamma^5)u(2)\right]$$

After averaging over initial/final spins, we find

$$\langle |\mathcal{M}_{fi}|^2 \rangle = 2\left(\frac{g_w}{M_w c}\right)^4 (p_1 \cdot p_2)(p_3 \cdot p_4)$$

which in the COM frame (neglecting electron mass)

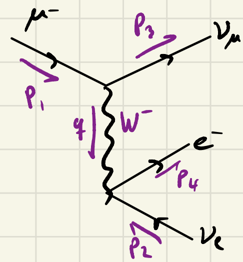
$$\langle |\mathcal{M}_{fi}|^2 \rangle = 8\left(\frac{g_W E}{M_W c^2}\right)^4 \left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right]$$

Using $\dfrac{d\sigma}{d\Omega} = \left(\dfrac{\hbar c}{8\pi}\right)^2 \dfrac{\langle |\mathcal{M}_{fi}|^2\rangle}{(2E)^2} \dfrac{|\vec{p}_\mu|}{|\vec{p}_e|}$ $\qquad \dfrac{|\vec{p}_\mu|}{|\vec{p}_e|} = \left[1-\left(\dfrac{m_\mu c^2}{2E}\right)^2\right]$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left[\frac{\hbar c\, g_W^2 E}{4\pi (M_W c^2)^2}\right]^2 \left[1-\left(\frac{m_\mu c^2}{2E}\right)^2\right]^2$$

implying $\sigma = \dfrac{d\sigma}{d\Omega} = \dfrac{1}{8\pi}\left[\dfrac{\hbar c\, g_W^2 E}{(M_W c^2)^2}\right]^2 \left[1-\left(\dfrac{m_\mu c^2}{2E}\right)^2\right]^2$.

The muon decay process is similar,



Momenta of final state particles are no longer fixed, so integrals are no longer trivial...

$$\Gamma = \left(\frac{m_\mu g_W}{M_W}\right)^4 \frac{m_\mu c^2}{(2\hbar(8\pi)^3} \Rightarrow \tau = \frac{1}{\Gamma} \;\ldots$$

Fermion Chirality

- Project spinors along helicity axes

$$u_L(p) = \tfrac{1}{2}(1-\gamma^5)\,u(p) \qquad \text{(not a helicity eigenstate)}$$

$$v_L(p) = \tfrac{1}{2}(1+\gamma^5)\,v(p)$$

and the corresponding right-handed spinors are

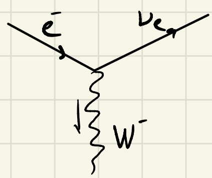
$$u_R(p) = \tfrac{1}{2}(1+\gamma^5)\,u_R(p)$$

$$v_R(p) = \tfrac{1}{2}(1-\gamma^5)\,v_R(p).$$

$P_{L/R} = \frac{1}{2}(1 \pm \gamma^5)$ projection operator; $P_{L/R}^2 = P_{L/R}$ ...

The adjoint chiral spinors are easily defined, $\bar{\omega}_{L,R}(p) = \bar{\omega}\,\frac{1}{2}(1 \overset{u}{\pm}/\overset{v}{\mp}\, \gamma^5)$

→ For charged weak current interactions,



the contribution to the amplitude A_{fi} from a leptonic vertex is then

$$j_\mu^- = \bar{\nu}\,\gamma_\mu\,\tfrac{1}{2}(1-\gamma^5)\,e$$

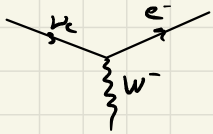
neutrino/electron spinors

$$= \bar{\nu}\,\tfrac{1}{2}(1+\gamma^5)\,\gamma_\mu\,\tfrac{1}{2}(1-\gamma^5)\,e \qquad \text{by projection principle..}$$

$$= \bar{\nu}_L\,\gamma_\mu\,e_L$$

which is now a purely vectorial index! Good.

We also have the opposite combination,



$$j_\mu^+ = \bar{e}_L \gamma_\mu \nu_L \quad \ldots$$

Notice that any opposite-handed spinor combination vanishes since $P_L P_R = 0$; $\overline{e_R} \gamma_\mu e_L = \bar{e} \gamma_\mu P_R P_L e = 0 ..$

Define the electromagnetic "current" as

$$j_\mu^{em} = -\bar{e}_L \gamma_\mu e_L - \bar{e}_R \gamma_\mu e_R$$

easy to show that $u_R + u_L = u$

minus sign conventional, represents electron charge...

Hence

$$\boxed{\begin{aligned} j_\mu^- &= \bar{\nu}_L \gamma_\mu e_L \\ j_\mu^+ &= \bar{e}_L \gamma_\mu \nu_L \end{aligned} \Bigg\} \text{ Charged weak currents} \qquad j_\mu^{em} = -\bar{e}_L \gamma_\mu e_L - \bar{e}_R \gamma_\mu e_R \Big\} \text{ Neutral EM current}}$$

Form Factors

Later...

Neutral Weak Interactions

Recall: isospin $\chi_1 = \begin{pmatrix}1\\0\end{pmatrix} = |p\rangle$ (up)

$\chi_2 = \begin{pmatrix}0\\1\end{pmatrix} = |n\rangle$ (down)

operator $\vec{\tau} = \frac{1}{2}\sigma_1\hat{e}_1 + \frac{1}{2}\sigma_2\hat{e}_2 + \frac{1}{2}\sigma_3\hat{e}_3$

with eigenstates $\pm\frac{1}{2}$, and eigenvalues of $\tau^2 = \frac{3}{4}$.

Isospin raising/lowering operators $\tau^{\pm} = \tau_1 \pm i\tau_2$.

Identical lie algebra to that of spin-$\frac{1}{2}$.

Weak currents: charged

We now can introduce lepton "doublets" $\chi \equiv \begin{pmatrix}e\\ \nu_e\end{pmatrix}$,

so that $\boxed{j_\mu^{\pm} = \bar{\chi}_L \gamma_\mu \tau^{\pm} \chi_L}$

We deem this "weak-isospin" instead of just isospin to describe leptonic doublets.

Neutral Weak Currents:

We anticipate a full weak-isospin symmetry, which would imply the existence of a third "current" corresponding to $\frac{1}{2}\tau^3$,

$$\boxed{j_\mu^3 = \bar{\chi}_L \gamma_\mu \frac{1}{2}\tau^3 \chi_L = \frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma_\mu e_L}$$

which would correspond to a neutral current.

$$J_\mu \sim (\cdots)\begin{pmatrix} \vdots & \vdots & \vdots & \vdots \end{pmatrix}\begin{pmatrix} \vdots \end{pmatrix} \quad \mu = 0,1,2,3$$

(Essentially, these are vertex factors... which are scalars: $\bar{u}\gamma_\mu u$ contributions; not sure why we call them "currents")
(their components are scalars; they're really 4-vectors)

Define now the "weak-hypercharge" current $j_\mu^Y \equiv 2j_\mu^{em} - 2j_\mu^3$,

$= \ldots$

$$\boxed{j_\mu^Y = -2\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L - \bar{\nu}_L\gamma_\mu \nu_L}$$

$Q \equiv I_3 + \frac{1}{2}Y \ldots$

which is invariant under the weak-isospin transformation

$$e_R \to e_R$$
$$e_L \to \nu_L$$
$$\nu_L \to e_L$$

which is good, because we can now express our currents for any weak-isospin doublet χ_L:

$$\chi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L \qquad \begin{array}{c} \text{weak-isospin } I_3 \\ \begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \end{array}$$

where the weak currents can be constructed as follows:

$$\boxed{\begin{aligned} \vec{j}_\mu &= \tfrac{1}{2}\bar{\chi}_L\gamma_\mu\vec{\tau}\chi_L \\ j_\mu^Y &= 2j_\mu^{em} - 2j_\mu^3 \end{aligned}}$$

Hypercharge is then given by $Y = 2Q - 2I_3$, which for leptons/quarks is

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \Rightarrow \begin{pmatrix} 2(0) - 2(\frac{1}{2}) \\ 2(-1) - 2(-\frac{1}{2}) \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \Rightarrow \begin{pmatrix} 2(2/3) - 2(1/2) \\ 2(-1/3) - 2(-1/2) \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1/3 \end{pmatrix}$$

GSW Model for Electroweak Mixing

→ The 3 weak-isospin currents couple with strength g_w to a weak iso-triplet of vector bosons W while the weak hypercharge current couples with strength $g'/2$ to an iso-singlet vector boson B.

$$-i\left[g_w \vec{j}_\mu \cdot \vec{W}^\mu + \frac{g'}{2} j_\mu^Y B^\mu\right]$$

$$\vec{j}_\mu \cdot \vec{W}^\mu = j_\mu^1 W^{1\mu} + j_\mu^2 W^{2\mu} + j_\mu^3 W^{3\mu}$$

$$= \underbrace{\frac{1}{\sqrt{2}} j_\mu^+ W^{+\mu} + \frac{1}{\sqrt{2}} j_\mu^- W^{-\mu}}_{\text{charged}} + \underbrace{j_\mu^3 W^{3\mu}}_{\text{neutral}} \qquad (*)$$

where $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2)$.

What's interesting now is that the couplings to $W^{\pm}$ can be read off immediately by the coefficients above in (*).

i.e. In $e^- \to \nu_e + W^-$, we couple W^- to j^-, so

$$-ig_w \frac{1}{\sqrt{2}} j^-_\mu W^{-\mu} = -i\frac{g_w}{2\sqrt{2}}\left[\bar{\nu}_e \gamma_\mu (1-\gamma^5) e\right] W^{-\mu}$$

which has vertex factor $-i\frac{g_w}{2\sqrt{2}}\gamma_\mu(1-\gamma^5)$ as before.

In the GSW model, the $\underbrace{W^3 \text{ and } B}_{\text{(neutral!)}}$ components mix to produce one massless combination A^μ (the photon) and one orthogonal massive combination (the Z^0):

$$A_\mu = B_\mu \cos\theta_w + W^3_\mu \sin\theta_w \qquad W^3_\mu = Z_\mu \cos\theta_w + A_\mu \sin\theta_w$$

$$Z_\mu = -B_\mu \sin\theta_w + W^3_\mu \cos\theta_w \qquad B_\mu = -Z_\mu \sin\theta_w + A_\mu \cos\theta_w$$

θ_w: weak mixing angle

Which then yields

$$-i\left[g_w \vec{j}_\mu \cdot \vec{W}^\mu + \frac{g'}{2} j^Y_\mu B^\mu\right] = -i\left\{\left[g_w \sin\theta_w j^3_\mu + \frac{g'}{2}\cos\theta_w j^Y_\mu\right] A^\mu + \left[g_w \cos\theta_w j^3_\mu - \frac{g'}{2}\sin\theta_w j^Y_\mu\right] Z^\mu\right\}$$

The connection between EM and weak couplings is that we know the EM coupling to be the EM current coupled to the photon field,

$$-ig_e j_\mu^{em} A^\mu$$

with $j_\mu^{em} = j_\mu^3 + \frac{1}{2} j_\mu^Y$, which implies that

$$g_w \cos\theta_w = g' \cos\theta_w = g_e$$

That is, the weak and electromagnetic coupling constants are <u>not</u> independent!

→ Note that for the Z^0 boson (see above eq^{n}):

$$g_w \cos\theta_w j_\mu^3 - \frac{g'}{2} \sin\theta_w j_\mu^Y = g_w \cos\theta_w j_\mu^3 - \frac{g'}{2} \sin\theta_w (2 j_\mu^{em} - 2 j_\mu^3)$$

$$= (g_w \cos\theta_w + g' \sin\theta_w) j_\mu^3 - g' \sin\theta_w j_\mu^{em}$$

$$= \left(\frac{g_e \cos^2\theta_w + g_e \sin^2\theta_w}{\sin\theta_w \cos\theta_w} \right) j_\mu^3 - \left(\frac{g_e \sin^2\theta_w}{\sin\theta_w \cos\theta_w} \right) j_\mu^{em}$$

$$= g_Z (j_\mu^3 - \sin^2\theta_w j_\mu^{em})$$

where $g_Z \equiv \dfrac{g_e}{\sin\theta_w \cos\theta_w}$

hence implying that

$$-i g_Z (j_\mu^3 - \sin^2\theta_w j_\mu^{em}) Z^\mu$$

is the coupling term, but what's the vertex factor?

We can simply just read off the couplings.

Very Straightforward for neutrinos, i.e.

$\nu \to \nu + Z^0$ is:

$$j^3_\mu = \frac{1}{2}\bar{\nu}_L \gamma_\mu \nu_L \cancelto{0}{- \frac{1}{2}\bar{e}_L \gamma_\mu e_L}$$

so $\quad -ig_Z\left(j^3_\mu - \sin^2\theta_w\, j^{em}_\mu\right) Z^\mu \Rightarrow -ig_Z\left(\frac{1}{2}\bar{\nu}\gamma_\mu \frac{1}{2}(1-\gamma^5)\nu\right) Z^\mu$

with vertex factor $\quad -i\frac{g_Z}{2}\gamma_\mu\left(\frac{1}{2} - \frac{1}{2}\gamma^5\right)$